Journal of Engineering Physics and Thermophysics, Vol. 75, No. 5, 2002

INVESTIGATION AND ELABORATION OF METHODS OF MULTICOLOR OPTICAL THERMOMETRY

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UDC 621.745.5.06./.07:536.5

The various complex mathematical transformations previously proposed for eliminating methodological errors of multicolor radiation pyrometry are mainly reduced to the determination of such combinations of wavelengths or corrections of registered brightness for the spectral distribution of the radiating capacity of the object being thermometered at which its equivalent radiating capacity is equal to unity. Mathematical formulas of the determining parameters of multicolor pyrometry of radiation have been obtained. Engineering techniques for calculating the critical values of the determining and adjustable parameters at which the application of multicolor optical thermometry is excluded because of large errors have been developed. Algorithms of a priori and a posteriori calibration systems of multicolor pyrometry of radiation of objects with practically any spectral distribution of the radiating capacity are proposed.

Methods of contactless pyrometry of radiation are the most suitable and often the only possible methods for measuring high temperatures, temperatures of remote, moving, and small-size objects, and in many other cases. At present, the methodological characteristics of optical measurements of temperature are largely determined by the methodological errors caused by the deviation from unity and occasional changes in the coefficients ε and τ. Elimination of their influence on measurement data is being given primary consideration in modern thermometry [1–5].

Any optical thermometer (pyrometer) measures the temperature of an object indirectly, by measuring its brightness having an unambiguous (determined by the thermodynamic laws of thermal radiation) relation to the temperature of any thermodynamically equilibrium radiation. If the thermal radiation is at thermodynamic equilibrium with the object being thermometered, then their temperatures are equal, $\varepsilon = 1$, and at $\tau = 1$ the energy pyrometers (brightness and radiation ones) provide measurements of the real temperature of objects. This also follows from the pyrometric equation generalizing the parameters of traditional (energy and two-color) and multicolor radiation pyrometry:

$$\frac{1}{T} - \frac{1}{S\,(S_c)} = \frac{\lambda\,(\lambda_e)}{c_2} \ln\left[\varepsilon\,(\varepsilon_e)\,\tau\,(\tau_e)\right] . \quad (1)$$

In the case of two-color pyrometry, $\varepsilon_e = \varepsilon_1/\varepsilon_2$. Therefore, the two-color radiation temperature is equal to the object temperature for thermodynamically equilibrium ($\varepsilon_1 = \varepsilon_2 = 1$) and "gray" ($\varepsilon_1 = \varepsilon_2 < 1$) radiation, at which $\varepsilon_e = 1$.

Table 1 shows the mathematical expressions of the determining parameters of (1), i.e., ε_e and λ_e derived for various methods of multicolor radiation pyrometry (MRP).

Analysis of Table 1 shows that with increasing number of operating waves the expressions for ε_e take on a more complex form, providing $\varepsilon_e = 1$ at a certain combination of wavelengths for any character of the function $\varepsilon = f(\lambda)$, including $\varepsilon_1 = \varepsilon_2 ... \varepsilon_n = 1$ and $\varepsilon_1 = \varepsilon_2 ... \varepsilon_n = \text{const} < 1$. Such a complication of the algorithms of obtaining and processing primary information and the change-over to multicolor pyrometry considerably widen the possibilities and the field of application of optical thermometry for measuring real temperatures.

It has been proved in [6] that in the validity range of the Wien law for thermometry ($\lambda T < 2000 \cdot 10^{-6}$ m·K) any logarithmic derivative of brightness with respect to the wavelength is a monotonic function of the thermodynamically equilibrium radiation temperature. Proceeding from this, an equation relating the three- and four-color radiation temperatures to the object temperature has been derived and expressions describing the spectral distributions of the ra-

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1062-0125/02/7505-1207$27.00

TABLE 1. Determining Parameters of Equation (1)

MRP method	ε_e	λ_e
2CRP	$\frac{\varepsilon_1}{\varepsilon_2}$	$\frac{1}{\lambda_1^{-1}-\lambda_2^{-1}}$
3CRP	$\frac{\varepsilon_1\varepsilon_3}{\varepsilon_2^2}$	$\frac{1}{\lambda_1^{-1}-2\lambda_2^{-1}+\lambda_3^{-1}}$
4CRP	$\frac{\varepsilon_1\varepsilon_4}{\varepsilon_2\varepsilon_3}$	$\frac{1}{\lambda_1^{-1}-\lambda_2^{-1}-\lambda_3^{-1}+\lambda_4^{-1}}$
6CRP	$\frac{\varepsilon_1\varepsilon_3\varepsilon_5^2}{\varepsilon_4\varepsilon_6\varepsilon_2^2}$	$\frac{1}{\lambda_1^{-1}-2\lambda_2^{-1}+\lambda_3^{-1}-\lambda_4^{-1}+2\lambda_5^{-1}-\lambda_6^{-1}}$
8CRP	$\frac{\varepsilon_1\varepsilon_4\varepsilon_6\varepsilon_7}{\varepsilon_2\varepsilon_3\varepsilon_5\varepsilon_8}$	$\frac{1}{\lambda_1^{-1}-\lambda_2^{-1}-\lambda_3^{-1}+\lambda_4^{-1}-\lambda_5^{-1}+\lambda_6^{-1}+\lambda_7^{-1}+\lambda_8^{-1}}$

TABLE 2. Critical Adjustable and Determining Parameters and Domains of Their Existence

MRP method	$\lambda_{a.cr}$	Domains of existence of $\lambda_{e.cr}$ and $\varepsilon_{e.cr}$
2CRP	–	$\varepsilon_1 > \varepsilon_2$
3CRP	$\frac{2}{\lambda_1^{-1}+\lambda_3^{-1}}$	$\varepsilon_e > 0$ at $\lambda_e > 0$
4CRP	$\frac{1}{\lambda_1^{-1}-\lambda_3^{-1}+\lambda_4^{-1}}$	or
6CRP	$\frac{1}{\lambda_1^{-1}+\lambda_3^{-1}-\lambda_4^{-1}+2\lambda_5^{-1}-\lambda_6^{-1}}$	$\varepsilon_e < 0$ at $\lambda_e < 0$
8CRP	$\frac{1}{\lambda_1^{-1}-\lambda_3^{-1}+\lambda_4^{-1}-\lambda_5^{-1}+\lambda_6^{-1}+\lambda_7^{-1}-\lambda_8^{-1}}$	

diating capacity $\varepsilon = \exp(b + k\lambda^a)$, $\varepsilon = k\lambda^a$, and $\varepsilon = k(1+\lambda)^a$ and calculated expressions for the wavelength values for these distributions, at which $S_c = T$, have been obtained. These mathematical transformations, as the other solutions, are directed to the search for relations between S_c and T and conditions, including the spectral distributions of ε or corrections of the spectral brightness of the object, under which $S_c = T$.

Analysis of (1) shows that $S_c = T$ only for $\varepsilon_e = 1$. Therefore, the familiar solutions are reduced to a problem which has not previously been formulated in explicit form — determination of such a combination of wavelength values of the coefficients correcting the spectral brightnesses at which for the spectral distribution of the radiating capacity of the object $\varepsilon_e = 1$. Such a formulation of the problem is concrete and most exactly reflects the potentialities and directions of the development of multicolor radiation pyrometry.

It is technically difficult and inexpedient to change the values of all operating wavelengths $\lambda_1 < \lambda_2 < ... < \lambda_n$. The same effect ($\varepsilon_e = 1$) can be obtained by changing the length of one adjustable wave λ_a with the aim of determining its optimal value $\lambda_{a.o}$ within the adjustable spectral range limited by the adjacent edge waves and in the case of three-color pyrometry — by the boundary waves. The character of change in ε_e with a change in λ_a depends on the spectral distribution of the object's radiating capacity. For example, in the case of thermodynamic equilibrium and "gray" radiation, ε_e does not change at all and remains equal to unity for any value of λ_a within the operating spectral range. The equivalent radiating intensity is always greater than zero; therefore, $\ln \varepsilon_e > 0$ at $\varepsilon_e > 1$, $\ln \varepsilon_e < 0$ at $\varepsilon_e < 1$, and $\ln \varepsilon_e = 0$ at $\varepsilon_e = 1$. Then $S_c = T$ and the methodological errors are equal to zero.

The adjustable spectral range and $\lambda_{a.o}$ in combination with the other working waves should provide:

1) $\varepsilon_e = 1$ for the spectral distribution of the object's radiating capacity;

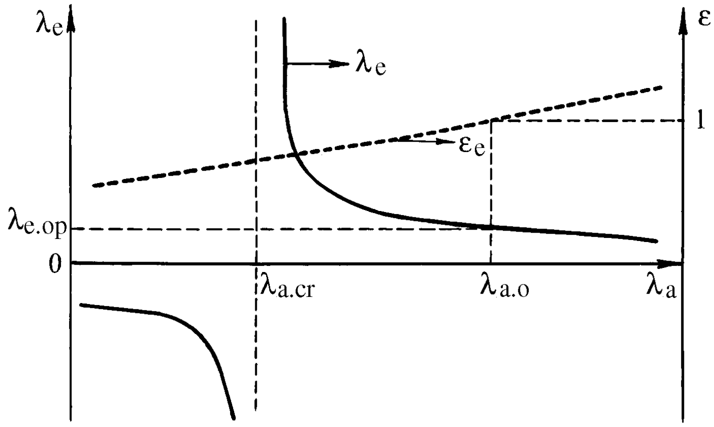

Fig. 1. Dependences of the equivalent wavelength and equivalent radiating capacity on the adjustable wavelength.

2) the least possible value of λ_e attained by making $\lambda_{a.o}$ differ from $\lambda_{a.cr}$ or, at least, by excluding coincidence of $\lambda_{a.o}$ with $\lambda_{a.cr}$.

For three-color radiation pyrometry, $\lambda_a = \lambda_2$ and the values of $\lambda_{a.o}$ and $\lambda_{a.cr}$ for the monotonic functions $\varepsilon = f(\lambda)$ are always within the adjustable spectral range equal to the operating spectral range. As the working wavelengths change, $\lambda_{a.o}$ and $\lambda_{a.cr}$ shift in one direction but with a different step ($\Delta\lambda_{a.cr} > \Delta\lambda_{a.o}$), which makes it possible to control the position of $\lambda_{a.o}$ and $\lambda_{a.cr}$. However, on objects with a spectral distribution described by the function $\varepsilon = f(\lambda) = \exp(\lambda^{-1}$ sec) optimum adjustment of MRP systems is, in principle, prohibitive, since at $\lambda \ln(\varepsilon) = c = \text{const}$ $\lambda_{a.o} = \lambda_{a.cr}$ and $\Delta\lambda_{a.o} = \Delta\lambda_{a.cr}$.

If $\varepsilon_e = 1$, then the absolute value of λ_e does not influence errors. However, for values of the adjustable wavelength equal to the critical values $\lambda_a = \lambda_{a.cr}$ the application of multicolor pyrometry is excluded because of the uncertainty caused by $\lambda_e = \infty$. Methodological errors of multicolor pyrometry sharply increase also in the case where the values of the parameters of Eq. (1) approach the critical values ($\lambda_e \to \lambda_{e.cr}$; $\varepsilon_e \to \varepsilon_{e.cr}$) that are found from the condition $\frac{1}{T} = \frac{\lambda_{e.cr}}{c_2} \ln \varepsilon_{e.cr}$ and hold in certain ranges of values of λ_e and ε_e.

Table 2 gives the mathematical expressions for the critical values of the adjustable wavelengths and determining parameters of multicolor thermometry and the boundaries of their existence domain. The expressions determining $\lambda_{a.cr}$ are given for the case where λ_2 is used as an adjustable wave.

Figure 1 shows the dependences of λ_e and ε_e on the adjustable wavelength for multicolor pyrometry at fixed values of the operating wavelengths for a decreasing spectral distribution of the radiating capacity. Analysis of the dependences shows that at $\lambda_a \to \lambda_{a.cr}$ $\lambda_e \to \infty$ and at $\lambda_a = \lambda_{a.o}$ $\varepsilon_e = 1$ and $\lambda_e = \lambda_{e.op}$.

With increasing number of operating waves the λ_e value increases, and then, even at slight deviations of ε_e from unity, methodological errors can considerably exceed allowable values. Deviations of ε_e can be caused by the limited spectral resolution of the systems of multicolor radiation pyrometry or the change in the spectral distribution of the radiating capacity of the object in the process of control. In these cases, as follows from Eq. (1), methodological errors are directly proportional to λ_e values. From the mathematical expressions given in Table 1 one can obtain a general equation defining the ways of decreasing λ_e:

$$\lambda_e = \frac{1}{\sum_{i=1}^{m} \lambda_i^{-1} - \sum_{i=1}^{n-m} \lambda_i^{-1}}.$$

Analysis of this equation and the above results shows that to obtain $\varepsilon_e = 1$ of the object and the least possible value of λ_e, it is necessary to meet the following requirements:

1) the medium waves, including $\lambda_{a.o}$, should be positioned in the portion of the spectral range with the maximum slope of the spectral distribution of the object's radiating capacity;

2) it is necessary to provide the largest possible width of the adjustable spectral range;

3) it is necessary to decrease the number of operating wavelengths for multicolor pyrometry;

4) it is necessary to maximally scatter, as to the spectrum, the boundary waves λ_1 and λ_n, i.e., position them, in the range, at the boundaries of the operating spectral range and not use them as adjustable waves;

5) the medium waves should be scattered towards the boundary waves so that one can obtain the greatest possible (equal to unity) difference between the number of added and subtracted waves in the short-wave and long-wave portions of the operating spectral range. In the range, it is necessary to shift all medium waves towards the right λ_n or, at least, the left boundary wave.

In minimizing λ_e, it is necessary to take into account that λ_1 is always "positive" and λ_n can be both "positive" and "negative." The waves in pairs λ_1–λ_2 and λ_{n-1}–λ_n have opposite signs. No more than two adjacent medium waves have equal signs.

The investigations performed have made it possible to develop a method of multicolor radiation pyrometry with a priori calculation of $\lambda_{a.o}$ for any spectral distributions of the radiating capacity of objects. For three-color radiation pyrometry of objects with a linear spectral distribution of ε, to decrease λ_e, the boundary waves λ_1 and λ_3 should be scattered, if possible, towards the boundaries of the operating spectral range. This possibility is determined from the conditions for decreasing λ_e at allowable deviations of ε_e from unity because of the change in the spectral distribution of the object's radiating capacity. In practice, deviations of ε_e increase with increasing difference between λ_1 and λ_3. The critical length of the adjustable wave is calculated by the formula for three-color pyrometry (Table 2). The spectral distribution of the radiating capacity of the chosen object is described by the function

$$\varepsilon = k\lambda + b\ .$$

By this function, ε_1 and ε_3 are determined for chosen λ_1 and λ_3. From the condition $\varepsilon_e = \varepsilon_1\varepsilon_3/\varepsilon_2^2 = 1$ one calculates ε_2, for which $\lambda_{a.o}$ is directly determined graphically by the spectral distribution of the radiating capacity or is calculated by the following expression derived with allowance for the above conditions:

$$\lambda_{a.o} = \frac{\sqrt{(k\lambda_1 + b)\ (k\lambda_3 + b)} - b}{k}\ .$$

If $\lambda_{a.o}$ coincides with $\lambda_{a.cr}$ or is close to it, then it is necessary to change λ_1 and λ_3 and repeat the calculation. In this case, to decrease λ_e, it is necessary to decrease λ_3.

By this technique, expressions for $\lambda_{a.o}$ and for other spectral distributions of the radiating capacity of objects have been obtained and the formulas for $\lambda_{a.o}$ for the function $\varepsilon = f(\lambda)$ previously deduced [6] by means of complicated mathematical transformations have been verified.

Technical realization of the proposed and familiar a priori methods requires preliminary knowledge of the spectral distribution of the object's radiating capacity. Therefore, these methods are mainly of theoretical interest for understanding and illustrating the potentialities of multicolor radiation pyrometry. A priori information on the spectral distribution of the radiating capacity makes it possible to measure the actual temperature of the object by means of both multicolor and traditional radiation pyrometry.

In the majority of technological objects, the initial spectral distribution of the radiating capacity changes in the process of measurements. For example, the function $\varepsilon = f/\lambda$ of carbonic iron melts under working conditions varies over a wide range under the action of many factors, including the holding time, mixing, the chemical composition and temperature of the metal, and the physicochemical processes associated with them [10–12].

For practical use, we have developed a method of multicolor radiation pyrometry with a posteriori calibration of the pyrometric system on objects. The point of the method is illustrated by the nomogram given in Fig. 2. The nomogram is intended for determining $\lambda_{a.o}$ of three-color radiation pyrometry with changes in the spectral distribution of the radiating capacity of melts under the action of temperature and its dependent physicochemical processes. To calibrate the pyrometric system, the dependences of methodological errors $\Delta = S_c - T$ on λ_a are taken for various temperatures uniformly overlapping the possible operating temperature range of the object. The next step is to determine

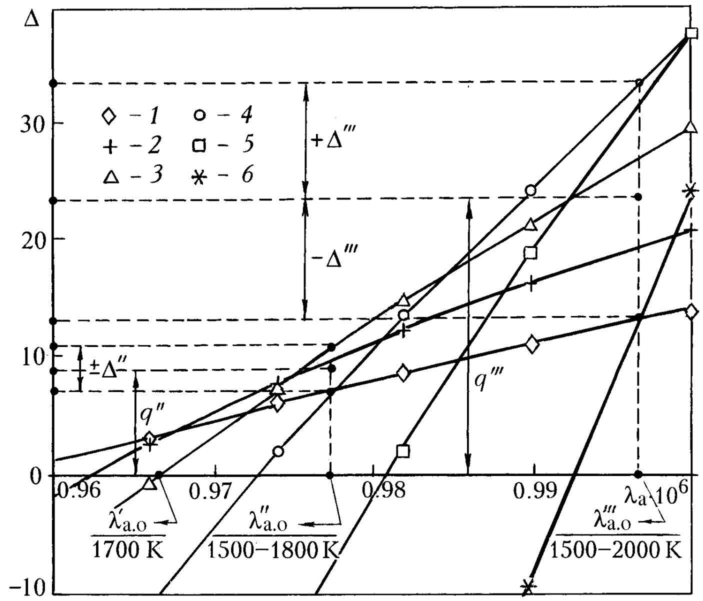

Fig. 2. Nomogram for determining $\lambda_{a.o}$ of three-color radiation pyrometry of carbonic iron melts for λ_1 = 0.7 μm; λ_3 = 1.2 μm; 1) T = 1500; 2) 1600; 3) 1700; 4) 1800; 5) 1900; 6) 2000 K. Δ, K; λ_a, m.

the coordinates of the shortest segment of the straight line perpendicular to the λ_a-axis connecting the $\Delta = f(\lambda_a)$ curves for the boundary temperatures of the temperature range needed for control. The projections of this segment on the λ_a- and Δ-axes determine, respectively, $\lambda_{a.o}$, the values of the constant correction q, and the methodological measurement errors. Under the conditions being considered this method makes it possible to:

1) decrease several times the methodological errors of optical thermometry of melts and, due to the exclusion of the systematic component because of the introduction of correction q''', decrease them down to the level of the remaining random errors $\Delta''' = \pm 10$ K in a wide technological temperature range from 1500 to 2000 K common for cast iron and steel;

2) considerably decrease the methodological errors in the technological temperature ranges, for example, of cast iron of 1500–1800 K down to $\Delta'' = \pm 2$ K;

3) practically exclude methodological errors in narrow temperature ranges often needed for technological control.

On the basis of the thermodynamic laws of thermal radiation and the pyrometric equation (1), it has been proved that the complicated mathematical transformations previously used for excluding methodological errors of multicolor pyrometry are reduced, in the final analysis, to the solution of a problem which was not formulated before — the determination of such values of wavelengths or corrections of spectral radiances for the spectral distribution of the object's radiating capacity at which its equivalent radiating capacity is equal to unity. Mathematical expressions of the determining parameters (equivalent radiating capacity and equivalent wavelength) of multicolor radiation pyrometry have been deduced. The existence of ranges of critical values of the determining parameters and adjustable wavelengths at which inadmissible high values of methodological errors exclude the employment of multicolor optical thermometry has been proved, engineering procedures for their computation have been developed, and these ranges have been limited. The application of multicolor pyrometry with a priori calibration has been extended to the region of practically any spectral distributions of the radiating capacity of objects for which calculation mathematical formulas have been derived, and recommendations for determining the optimum values of adjustable wavelengths providing a radiating capacity equal to unity at the least possible value of the equivalent wavelength have been worked out. Algorithms of a priori and a posteriori calibration of the systems of multicolor radiation pyrometry of objects with known and unknown spectral distributions of the radiating capacity, respectively, have been developed.

NOTATION

T, real temperature of the object, K; $S(S_c)$, brightness, radiation (color) temperatures of radiation, K; $\lambda(\lambda_e)$, equivalent (effective) wavelengths for energy (color) radiation pyrometry, m; c_2, second Planck constant, K·m; $\varepsilon(\varepsilon_e)$, radiating (equivalent radiating) capacity of the object; $\tau(\tau_e)$, transmission coefficient (equivalent coefficient) of the intermediate medium; λ_a, adjustable wavelength, m; $\lambda_{a.o}$, optimum wave length; $\lambda_{a.cr}$, critical adjustable wavelength; $\lambda_{e.cr}$, critical equivalent wavelength; $\lambda_{e.op}$, operating equivalent wavelength; $\varepsilon_{e.cr}$, critical equivalent radiating capacity; c, power coefficient; m, number of added wavelengths in the expression for calculating λ_e (Table 1); n, number of wavelengths used for calculating λ_e; $\sum_{i=1}^{m} \lambda_i^{-1}$, sum of inverse values of added wavelengths in the expression for λ_e, m^{-1}; $\sum_{i=1}^{n-m} \lambda_1^{-1}$, same for subtracted wavelengths; k, multiplicative coefficient, m^{-1}; b, dimensionless additive coefficient; a, dimensionless power coefficient; $\lambda'_{a.o}$, optimum adjustable wavelength in measuring temperatures of carbonic iron melts in the 1700 ± 10 K range; $\lambda''_{a.o}$, same for the 1500–1800 K range; $\lambda'''_{a.o}$, same for the 1500–2000 K range; Δ'', errors of temperature measurements of carbonic iron melts in the 1500–1800 K range; Δ''', same for the 1500–2000 K range; q'', value of the temperature correction in measuring temperatures of carbonic iron melts in the 1500–1800 K range; q''', same for the 1500–2000 K range. Subscripts: e, equivalent; c, color; a, adjustable; a.o, adjustable optimum; a.cr, adjustable critical; e.cr, equivalent critical; a.op, equivalent operating.

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